Multivariate Calculus Quiz #12
Iterated Integrals & Volume
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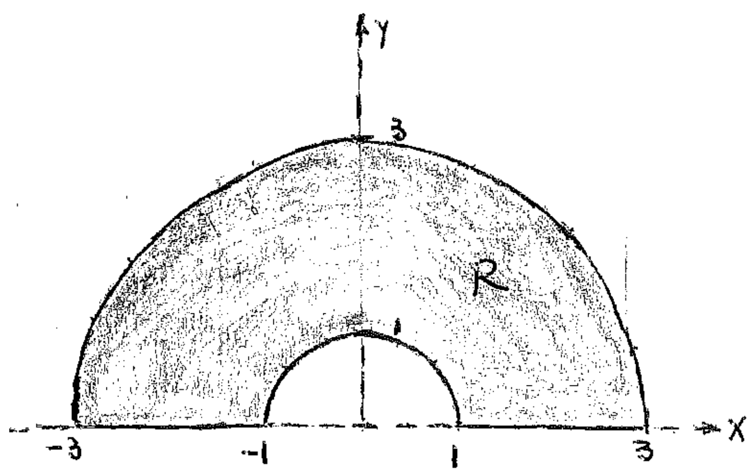
Instructions: Solve each of the following problems showing your steps. A calculator is not permitted for this quiz. You may do your work on a separate sheet(s) of paper.

1. Let R be the region bounded by (only) the two curves $y = 2x$ and $x = 4y - y^2$.
   a. (2 Pts) Sketch the region.
   b. (4 Pts) Set up an iterated integral and evaluate the area of this region.

2. (4 Pts) Let R be the region bounded by $y = x, y = 2$, and $x = 0$. Let $f(x, y) = x^2 + y^2$. Find the volume of the 3-D region (solid) between the surface $z = f(x, y)$ and the xy-plane over the region R. See pic in book: paraboloid over square region.

3. (4 Pts) Let R be the region illustrated below, and let $z = 9 - x^2 - y^2$. Find the volume of the 3-D region (solid) between the surface $z = f(x, y)$ and the xy-plane over the region R.

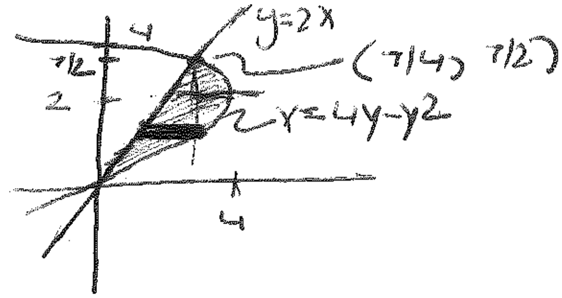


CALC 3 QUIZ soln

1. $y = 2x \qquad x = 4y - y^2 = y(4-y)$

a. (2pts) Sketch ~~the~~ region

−1 for $y=4$ as top y-limit



b. (4pts) set up an iterated integral and evaluate the area of this region.

$4y - y^2 = y/2 \quad \times 2$

$8y - 2y^2 = y$

$-y \qquad -y$

$7y - 2y^2 = 0$

$y(7-2y) = 0$

$y = 0$

$7 - 2y = 0$

$7 = 2y$

$y = 7/2$

$x = y/2$

$x = \frac{7/2}{2} = 7/4$

$$A = \iint_R dA = \int_0^{7/2}\int_{y/2}^{4y-y^2} dx\,dy$$

$4y - \frac{y}{2}$

$\frac{8y}{2} - \frac{y}{2} = \frac{7y}{2}$

$$A = \int_0^{7/2} x \Big|_{y/2}^{4y-y^2} dy = \int_0^{7/2} (4y - y^2 - y/2)\,dy$$

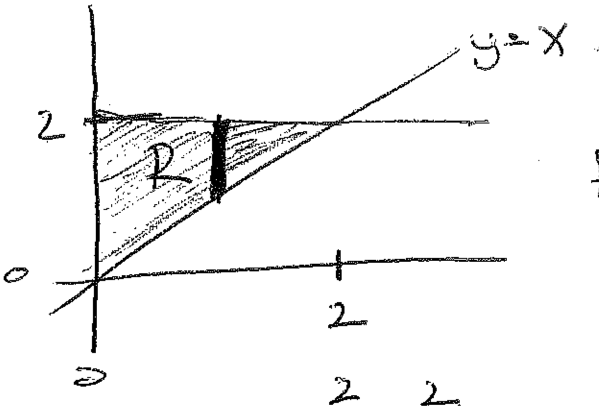
$$A = \int_0^{7/2} \left(\frac{7y}{2} - y^2\right) dy = \left[\frac{7y^2}{4} - \frac{y^3}{3}\right]_0^{7/2}$$

$$A = \frac{7}{4}\left(\frac{7}{2}\right)^2 - \frac{1}{3}\left(\frac{7}{2}\right)^3 = \cancel{\left(\frac{7}{2}\right)^2\left[\frac{7}{4} - \frac{1}{3}\cdot\frac{7}{2}\right]}$$

$$= \frac{1}{2}\cdot\frac{7^3}{2^3} - \frac{1}{3}\cdot\frac{7^3}{2^3} = \frac{7^3}{2^3}\left(\frac{1}{2} - \frac{1}{3}\right) = \left(\frac{7}{2}\right)^3\left(\frac{3-2}{6}\right)$$

$$A = \frac{343}{8}\cdot\frac{1}{6} = \boxed{\frac{343}{48}} \approx 7.146$$

3 (4 pts)



$f(x,y) = x^2 + y^2$

$$V = \iint_R f(x,y)\,dA = \int_0^2 \int_x^2 (x^2 + y^2)\,dy\,dx$$

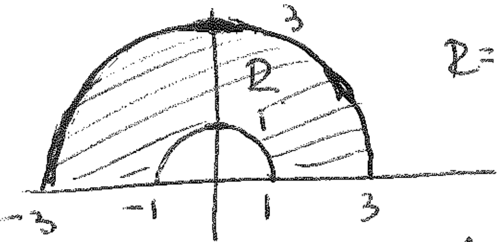
$$V = \int_0^2 \left[x^2 y + \frac{y^3}{3} \right]_x^2 dx = \int_0^2 \left[2x^2 + \frac{8}{3} - \left(\frac{3x^3}{3} + \frac{x^3}{3} \right) \right] dx$$

$$V = \int_0^2 \left[2x^2 + \frac{8}{3} - \frac{4x^3}{3} \right] dx = \left[\frac{2x^3}{3} + \frac{8}{3}x - \frac{\cancel{4}x^4}{3\cdot\cancel{4}} \right]_0^2$$

$$V = \left[\frac{2x^3}{3} + \frac{8}{3}x - \frac{x^4}{3} \right]_0^2 = \frac{1}{3} \left[2x^3 + 8x - x^4 \right]_0^2$$

$$V = \frac{1}{3} \left[2^4 + \cancel{16} - \cancel{16} - 0 \right] = \boxed{16/3}$$

\#3 (4pts)



$R = \{(r,\theta) : 1 \le r \le 3,\ 0 \le \theta \le \pi\}$

$z = 9 - r^2$

$$\boxed{V = \int_0^{\pi}\int_1^{3} (9-r^2)\, r\, dr\, d\theta} = \int_0^{\pi}\int_1^{3} (9r - r^3)\, dr\, d\theta$$

$$= \int_0^{\pi} \left[\frac{9r^2}{2} - \frac{r^4}{4} \right]_1^3 d\theta = \frac{1}{2}\int_0^{\pi} \left[9r^2 - \frac{r^4}{2} \right]_1^3 d\theta$$

$$\frac{1}{2}\int_0^{\pi} \left[81 - \frac{81}{2} - \left(9 - \frac{1}{2}\right) \right] d\theta = \frac{1}{2}\int_0^{\pi} \left[\frac{162-81}{2} - \left(\frac{18-1}{2}\right) \right] d\theta$$

$$V = \frac{1}{2}\int_0^{\pi} \left[\frac{81}{2} - \frac{17}{2} \right] d\theta = \frac{1}{2}\int_0^{\pi} \frac{64}{2}\, d\theta = 16\int_0^{\pi} d\theta$$

$$= \boxed{16\pi}$$